THERMALLY-CONSTRAINED FUEL-OPTIMAL ISS MANEUVERS

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Optimal Propellant Maneuvers (OPMs) are now being used to rotate the International Space Station (ISS) and have saved hundreds of kilograms of propellant over the last two years. The savings are achieved by commanding the ISS to follow a pre-planned attitude trajectory optimized to take advantage of environmental torques. The trajectory is obtained by solving an optimal control problem. Prior to use on orbit, OPM trajectories are screened to ensure a static sun vector (SSV) does not occur during the maneuver. The SSV is an indicator that the ISS hardware temperatures may exceed thermal limits, causing damage to the components. In this paper, thermally-constrained fuel-optimal trajectories are presented that avoid an SSV and can be used throughout the year while still reducing propellant consumption significantly.

INTRODUCTION

This paper introduces fuel-optimal attitude maneuvers for the International Space Station (ISS) that satisfy a thermal requirement designed to protect ISS hardware. On August 1, 2012, the first flight demonstration of an Optimal Propellant Maneuver (OPM) rotated the ISS by 180 deg using only 9.7 kg of propellant[1]. An identical maneuver performed in the same amount of time without OPM would typically have used about 50-150 kg, depending on the jet selection. Since then, two dozen OPMs have together saved over 1500 kg of propellant. For a current cost-to-orbit of over $20,000/kg[‡‡], the propellant savings add up to more than $30 million. Moreover, the fewer thruster firings during OPMs mean less stress on the ISS structure compared to standard maneuvers.

The OPM method uses a pre-planned attitude command trajectory to transition the ISS between specified rotational states. The trajectory is designed to take advantage of the complete nonlinear system dynamics to reduce propellant usage. Thus it differs from the eigenaxis maneuver, the shortest kinematic path between two orientations, and has a non-constant rotation rate. An optimal control problem is solved to obtain the attitude maneuver commands. Any physical, hardware, software, or operational limits can be included as constraints. For example, the maneuver time is restricted to one orbital period (about 93 minutes for ISS). Moreover, slow maneuver

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rates can result in a static sun vector (SSV), exposing the ISS to excessive sunlight which may cause hardware temperatures to exceed thermal limits. Therefore, it is desired that the average rate of change of the sun vector must not fall below a specified threshold during the maneuver. In this paper, a newly-designed TCOPM (Thermally-Constrained Optimal Propellant Maneuver) is described that ensures no thermal threats to the ISS structure occur during the entire attitude maneuver while still minimizing the propellant usage.

In the following sections, the ISS-Earth-Sun geometry is illustrated with some definitions, the ISS thermal requirement to avoid a static sun vector is explained, and maneuvers satisfying the requirement are presented. Note that the ISS thermal requirement is conservatively formulated for ease of analysis. If the analysis reveals that the TCOPM attitude commands steer clear of an SSV, then the extensive additional thermal analysis will not be necessary. The results demonstrate it is possible to protect against an SSV during a fuel-optimal maneuver.

THE THERMAL ENVIRONMENT

We first establish some preliminaries in order to characterize the ISS thermal environment. Define the Earth-Centered Inertial (ECI) frame to have the positive x-axis point toward the vernal equinox and the positive z-axis point toward the North Pole, with the y-axis completing the right hand rule.[2] The Earth's equatorial plane (ECI x-y plane) is tilted from Earth's orbital plane by $\varepsilon = 23.4°$. Then the unit vector pointing towards the sun is given by

$$\hat{s} = \begin{bmatrix} \cos\Gamma \\ \sin\Gamma\cos\varepsilon \\ \sin\Gamma\sin\varepsilon \end{bmatrix} \quad (1)$$

where Γ is the angular separation of the Earth from the vernal equinox (the ecliptic true solar longitude).[3,4]

It is also useful to define the solar beta angle, the angle between the ISS orbit plane and the sun vector, $\hat{s}$, as depicted in Figure 1. The inclination of the ISS orbit plane is $i = 51.6°$ as measured from the equatorial plane. Let Ω, the longitude of the ascending node, be the angle (measured in the equatorial plane) between the ECI x-axis and the point where the ISS crosses the equatorial plane in a northerly direction.[2] Then the unit vector, $\hat{n}$, normal to the ISS orbit plane is given by

$$\hat{n} = \frac{\mathbf{r}_{ISS} \times \mathbf{v}_{ISS}}{\|\mathbf{r}_{ISS} \times \mathbf{v}_{ISS}\|} = \begin{bmatrix} \sin\Omega\sin i \\ -\cos\Omega\sin i \\ \cos i \end{bmatrix} \quad (2)$$

where $\mathbf{r}_{ISS}$ and $\mathbf{v}_{ISS}$ are the translational position and velocity of the ISS in the ECI frame.[3] Then the solar beta angle[4,5] is defined as

$$\beta = \sin^{-1}(\hat{s} \cdot \hat{n}) = \sin^{-1}(\cos\Gamma\sin\Omega\sin i - \sin\Gamma\cos\varepsilon\cos\Omega\sin i + \sin\Gamma\sin\varepsilon\cos i) \quad (3)$$

When the solar beta angle is large in magnitude, the sun vector is nearly perpendicular to the ISS orbit plane, and thus the ISS is exposed to much more sunlight per orbit, and one side of the ISS experiences significant sun exposure. In contrast, a small solar beta angle indicates the sun vector nearly lies in the ISS orbit plane so there are long periods when the Earth is between the ISS and the Sun. In that case, the sun exposure over an orbit is more evenly distributed across the entire

ISS. As the Earth orbits the Sun and as the ISS orbit precesses due to the Earth's oblateness, the solar beta angle varies throughout the year, as shown in Figure 2. Notice that $\Gamma = 90°$ and $\Omega = 180°$ produces the beta angle

$$\beta = \sin^{-1}(\cos\varepsilon \sin i + \sin\varepsilon \cos i) = \varepsilon + i = 75° \tag{4}$$

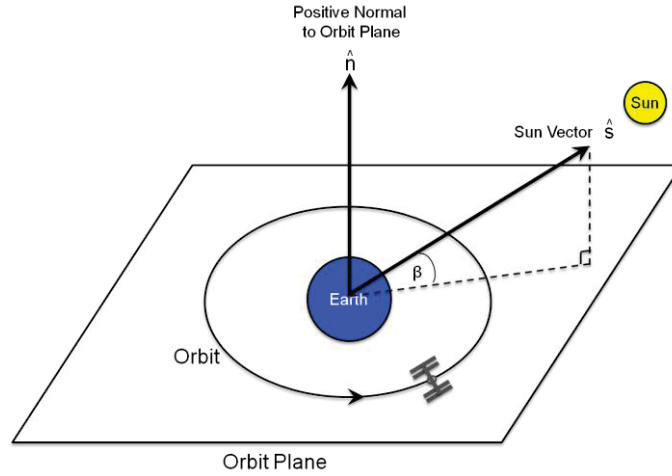which is the maximum solar beta angle that is possible for the ISS.

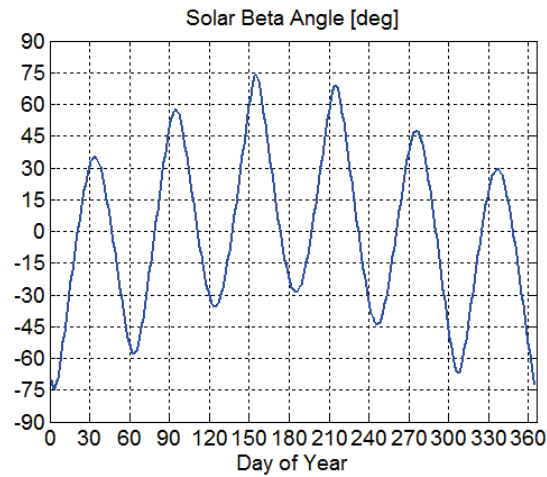

Figure 1. Solar beta angle definition.

Figure 2. Solar beta angle variation over one year.

The fraction of an orbit when the ISS is eclipsed by Earth's shadow, f_E, can be computed from the solar beta angle as follows. Assuming a circular orbit and a cylindrical shadow cast by the Earth[5]

$$f_E = \frac{1}{\pi}\cos^{-1}\left(\frac{\cos\beta^*}{\cos\beta}\right) = \frac{1}{\pi}\cos^{-1}\left(\frac{\sqrt{1-R^2/(R+h)^2}}{\cos\beta}\right) \qquad (5)$$

where R is the Earth radius, h is the ISS altitude, and β^* is the critical angle

$$\beta^* = \sin^{-1}\left(\frac{R}{R+h}\right), \qquad 0° \le \beta^* \le 90° \qquad (6)$$

as Figure 3 illustrates (adapted from Reference 5). When the solar beta angle magnitude exceeds β^* (vertical lines in plot), the ISS is always in sunlight ($f_E = 0$). Assuming an ISS altitude of
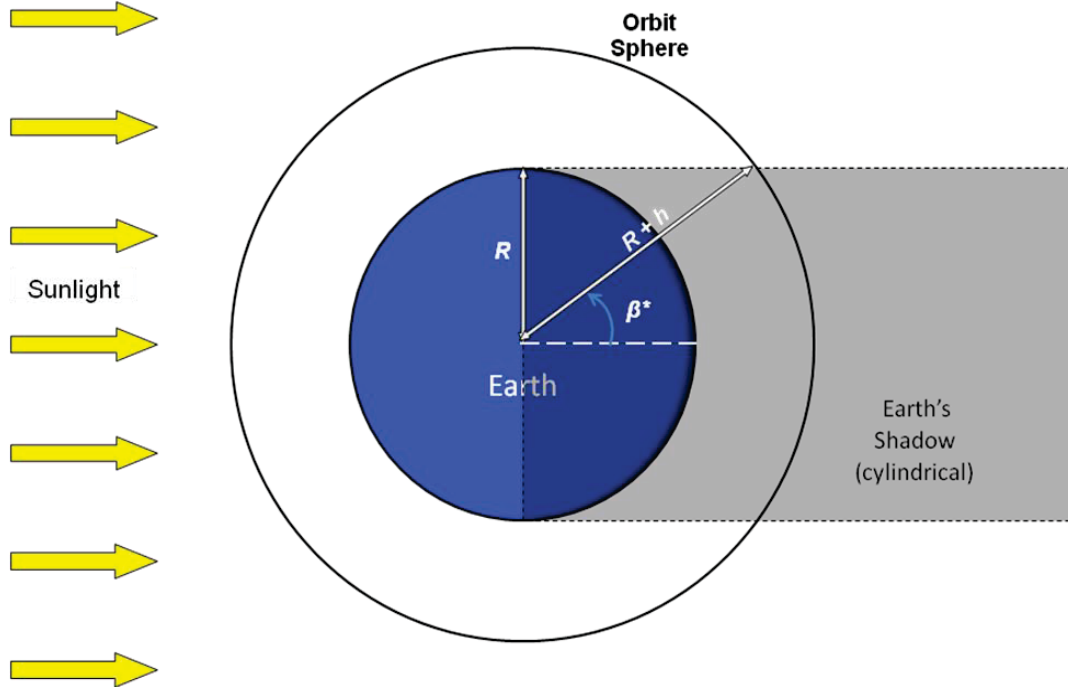

Figure 3. Eclipsing of a circular orbit and critical angle.

415 km, the critical angle is $\beta^* = 69.9°$. The ISS eclipse fraction is plotted as a function of the solar beta angle in Figure 4. The longest period of eclipse, nearly two-fifths of an orbit, happens only when the solar beta angle is zero.

Finally, a simple test using the ISS and Sun positions can be used to determine whether the ISS is in eclipse. Let

$$\phi = \cos^{-1}\left(\frac{-\mathbf{r}_{ISS}^T(\mathbf{r}_{Sun} - \mathbf{r}_{ISS})}{\|\mathbf{r}_{ISS}\|\|\mathbf{r}_{Sun} - \mathbf{r}_{ISS}\|}\right) \qquad (7)$$
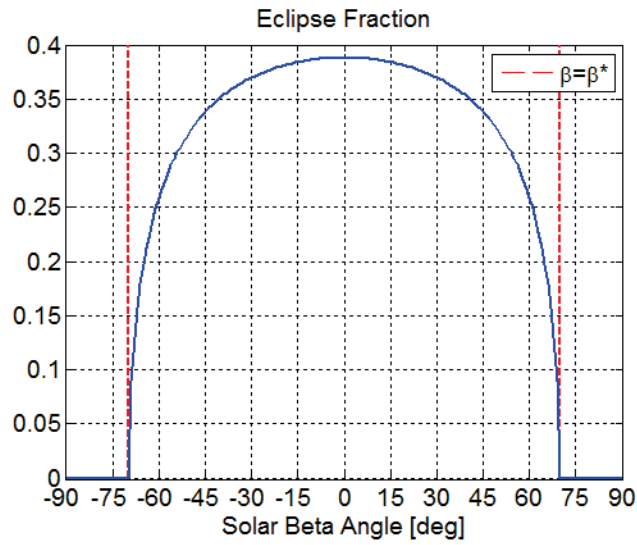
Figure 4. Fraction of orbit spent in eclipse versus solar beta angle.

be the angle between the ISS-to-Earth and the ISS-to-Sun vectors. Then the ISS falls in Earth's shadow if $\phi < 90°$ and $\left\| \mathbf{r}_{ISS} \right\| \sin \phi < R$. For example, the ISS is shown in three different positions in Figure 5, and only ϕ_1 would indicate the ISS is in shadow.
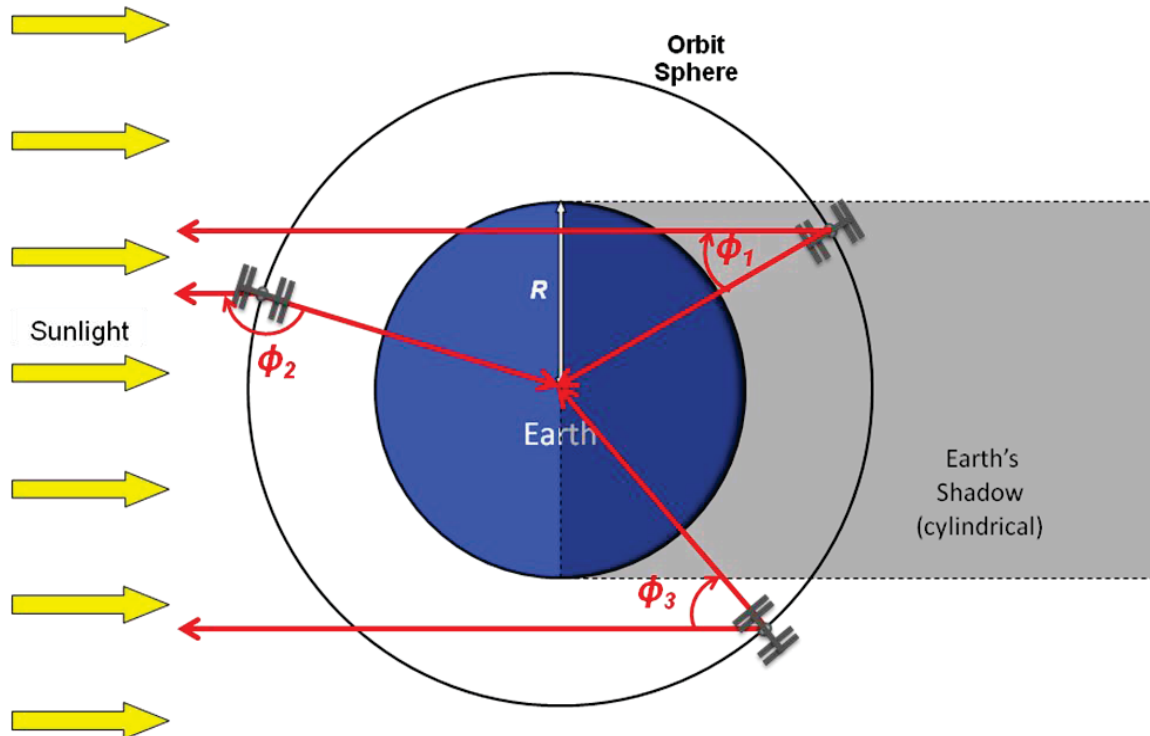

Figure 5. Determining if the ISS is in shadow.

THE THERMAL REQUIREMENT

ISS hardware components can be damaged by excessive exposure to direct sunlight, which is the primary thermal concern. Cold temperatures during eclipse and the susceptibility of the truss structure to thermal-gradient-induced loads exceedances (due to self-shadowing) are also of concern, but are out of the scope of this paper. Thus the ISS thermal requirement under consideration focuses solely on insolation of ISS hardware. Long insolation periods for an ISS component can be indicated by the sun vector expressed in the ISS body reference frame remaining static as the ISS rotates during a maneuver. The sun vector is considered static when it falls within a 16.7 deg half-cone relative to the ISS body axes for 20 minutes cumulative time during the maneuver. Although this requirement is only meant to keep the Sun off of temperature-sensitive hardware, it is convenient to be conservative and apply it across the entire ISS. Therefore, it is preferred that the sun vector change with a 20-minute average rate greater than 33.4deg/20min = 1.67 deg/min to prevent an SSV on any given spot. If this more stringent test is passed, no further thermal analysis is necessary. If not, the maneuver must be carefully scrutinized to see if the SSV would actually impact the sensitive hardware.

The typical control reference frame for ISS is the Local Vertical Local Horizontal (LVLH) reference frame, which completes one rotation per orbit about the orbit normal vector. Thus while maintaining ISS attitude hold for one orbit (approximately 90 min), the Sun moves at a rate of roughly -4 deg/min about the y-axis in an LVLH reference frame. Consequently, performing a pitch maneuver starting with the ISS body aligned with LVLH and using a maneuver rate magnitude between 2.33 and 5.67 deg/min would cause a stagnant Sun condition. Traditionally, maneuvers are executed using an eigenaxis trajectory with an acceptable constant rate, but this approach may rapidly deplete the supply of propellant. The recent development of OPMs provides an alternative to standard eigenaxis maneuvers. In addition to saving propellant, OPMs take advantage of the natural torques on ISS to fire thrusters less frequently, thereby mitigating ISS structural loads in comparison to eigenaxis maneuvers. Decreased propellant consumption and increased ISS structural life have long-term benefits to the ISS program. Consequently, it is desirable to use OPMs as often as possible. Unlike an eigenaxis maneuver which follows the shortest kinematic trajectory, an OPM exploits path dependence to minimize fuel consumption. Thus the nature of OPM and its non-uniform attitude and rate profile make thermal analysis more difficult.

For the first flight demonstrations of OPMs[1], the trajectories were not designed with the thermal specification in mind but were screened prior to flight to ensure the safety of ISS hardware. One such example OPM is presented in Figure 6. In this case, the maneuver transitions the ISS body from having the positive x-axis pointing along the velocity vector (a +XVV attitude) to having the negative x-axis pointing along the velocity vector (a –XVV attitude), essentially a pure 180 deg yaw rotation. The attitude profile for the OPM is clearly not a pure yaw rotation; large roll and pitch attitude excursions can be seen in the ISS attitude plot. But the maneuver took only 9.7 kg of propellant to complete, a 90% reduction in fuel cost. The sun vector in the ISS body frame is also shown along with the sun vector rate, which is nearly zero between about 20 to 30 min and 65 to 75 min into the maneuver. The sun rate magnitude is also plotted (green line), from which the 20 min forward average can be computed. Note that a static sun vector during eclipse is not a thermal concern, and so the average sun rate magnitude (blue line) is only plotted over 20 min intervals that are entirely sunlit. Despite excluding the eclipse period, the average sun rate magnitude still falls below the 1.67 deg/min threshold (red dashed line) and thus indicates an SSV twice during the maneuver (once before and once after the eclipse). It is this behavior that we wish to eliminate.
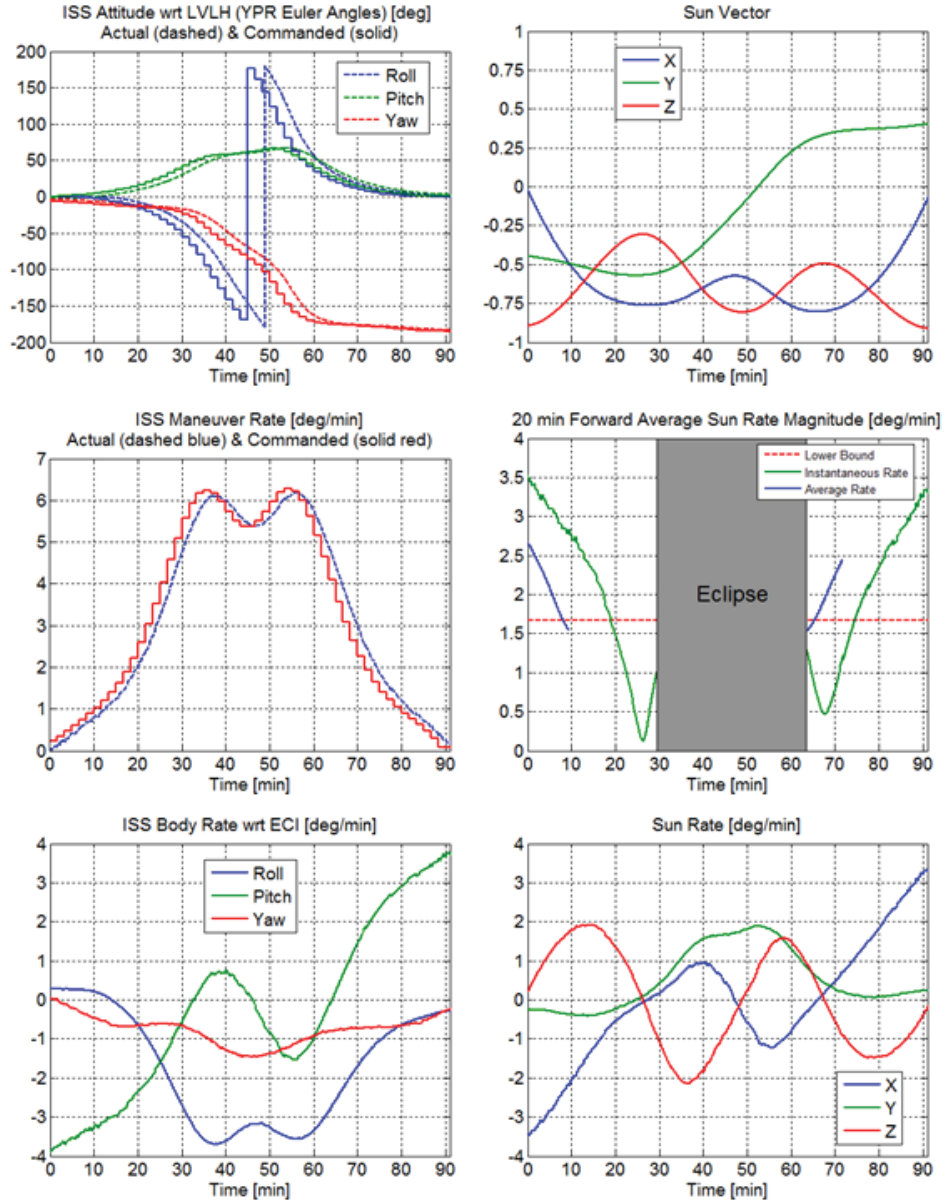
Figure 6. +XVV to –XVV trajectory using initial OPM (command set #1). The average sun rate magnitude falls below the preferred lower bound, indicating a static sun vector.

THERMALLY-CONSTRAINED FUEL-OPTIMAL MANEUVERS

Using the previously described framework[1], it is possible to design OPMs that satisfy the thermal specification. The tradeoff is that fewer candidate trajectories are feasible and so it may potentially require more propellant to complete the maneuver while avoiding an SSV. However, note that disproportionate insolation is the primary concern, so it is only necessary to satisfy the sun vector rate limit when the ISS is sunlit. Candidate trajectories that only exceed the rate limit during eclipse are viable since an SSV in Earth's shadow is not problematic. By initiating such a maneuver at ISS orbit noon, we can place an SSV near orbit midnight when it would be harmless

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to the ISS. We have recently created such Thermally-Constrained OPMs (TCOPMs), presented next.

Figure 7 demonstrates a TCOPM (referred to as +XVV to –XVV command set#2) that can perform the identical 180 deg yaw maneuver as in Figure 6 (+XVV to –XVV command set#1), but without exposing the ISS to a stagnant sun condition. The plots reveal that the only period with a static sun vector occurs during eclipse, as desired. In this case, the TCOPM took 10.8 kg of propellant to reorient the ISS, merely 1.1 kg more than command set #1.
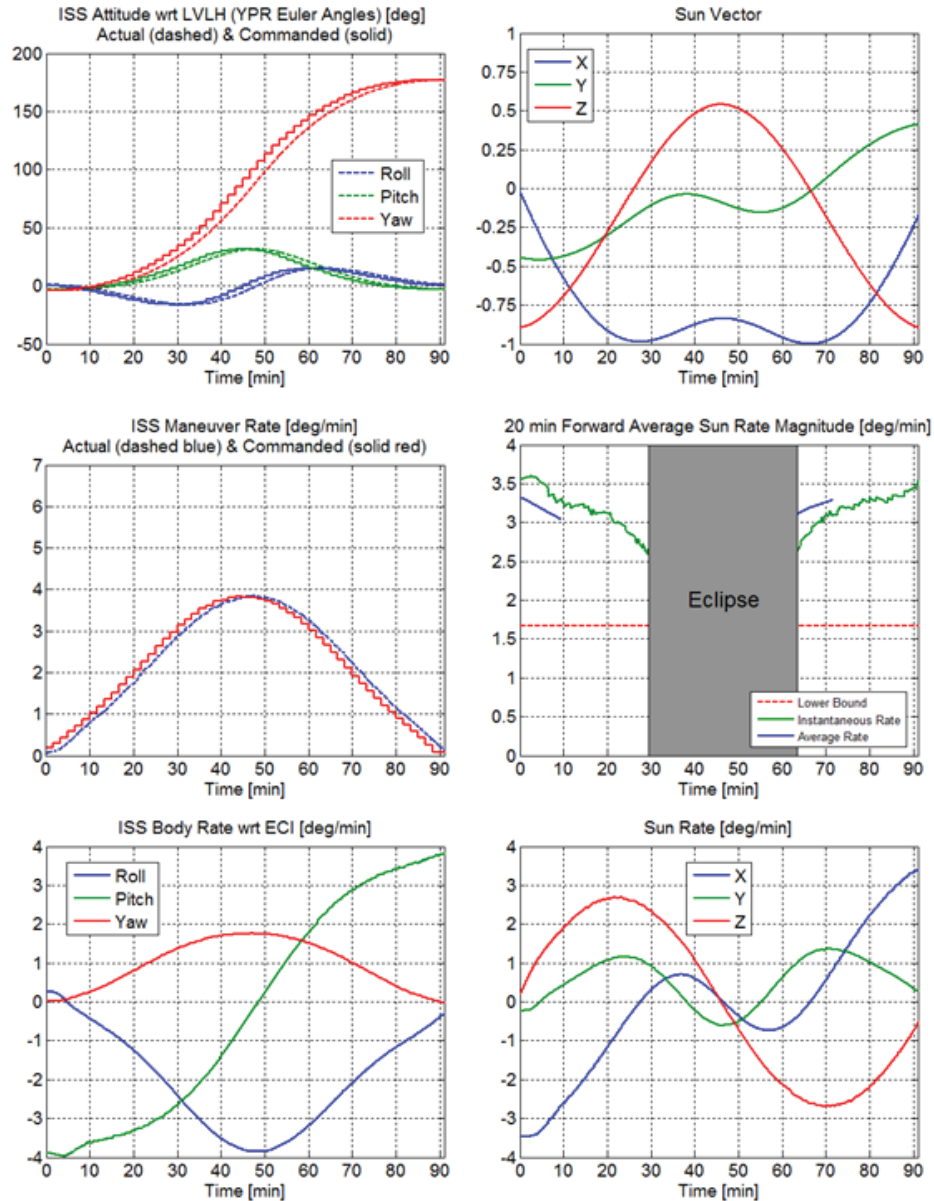

Figure 7. +XVV to –XVV trajectory using Thermally-Constrained OPM (command set #2). The average sun rate magnitude never approaches the lower limit.

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Similarly, the first OPM for the reverse maneuver[1], referred to here as –XVV to +XVV command set #1, also results in an SSV. Consequently, a corresponding TCOPM (command set #2) was designed as well (Figure 8). As with the +XVV to –XVV maneuver, the –XVV to +XVV OPM propellant consumption is nearly identical with or without the thermal constraint.
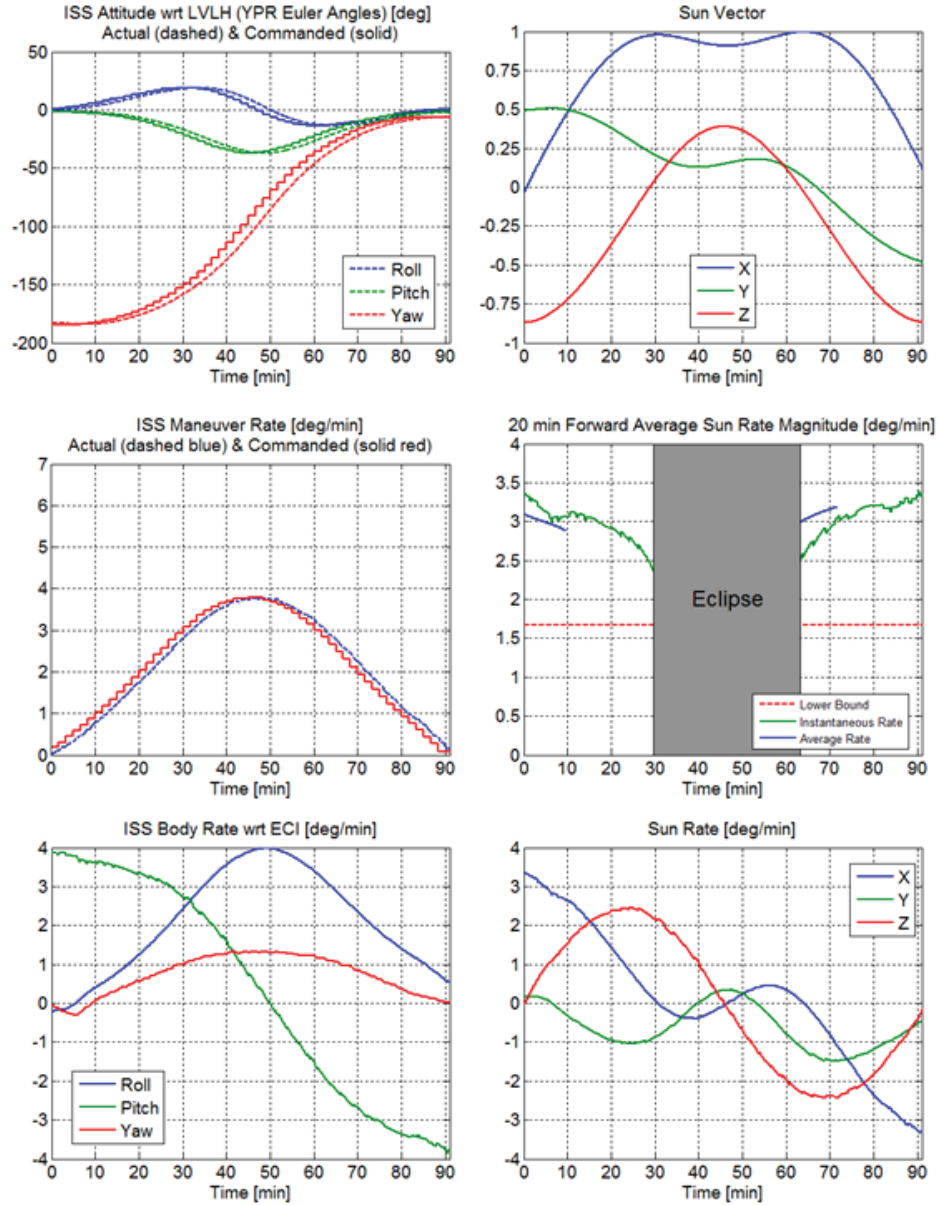

Figure 8. –XVV to +XVV trajectory using Thermally-Constrained OPM (command set #2). The average sun rate magnitude never approaches the lower limit.

Now that ISS assembly is complete and the ISS configurations and operations have finalized, one big advantage of OPMs is their robustness – the same fuel-optimal trajectory can be used repeatedly. Since the ISS environment changes throughout the year, it is necessary to assess TCOPM performance for different flight conditions to gain confidence for routine use. Prior to on-orbit execution, the sensitivity of each OPM to off-nominal initial conditions, mass properties,

and parameter uncertainty is analyzed via Monte Carlo simulations[1] using the high-fidelity Space Station Multi-Rigid Body Simulation (SSMRBS)[6]. Of particular interest for this paper, the TCOPM trajectories were simulated over a range of solar beta angles to envelope the ISS thermal environment. Figure 9 and **Figure 10** demonstrate the vast improvement that the TCOPMs provide over the original OPMs. For both command set #1 trajectories (+XVV to –XVV or the reverse), an SSV occurs for a large range of solar beta angles. In contrast, the TCOPMs maintain average sun rates within the acceptable range for all solar beta angles typically seen for ISS operations.
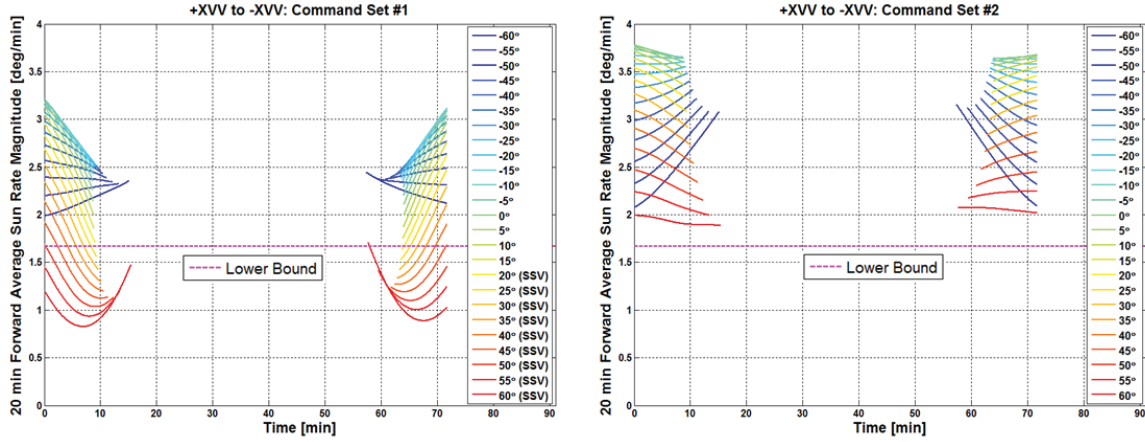

Figure 9. Comparison of average sun rate magnitude for various solar beta angles for +XVV to –XVV command set #1 (left) and the TCOPM command set #2 (right).
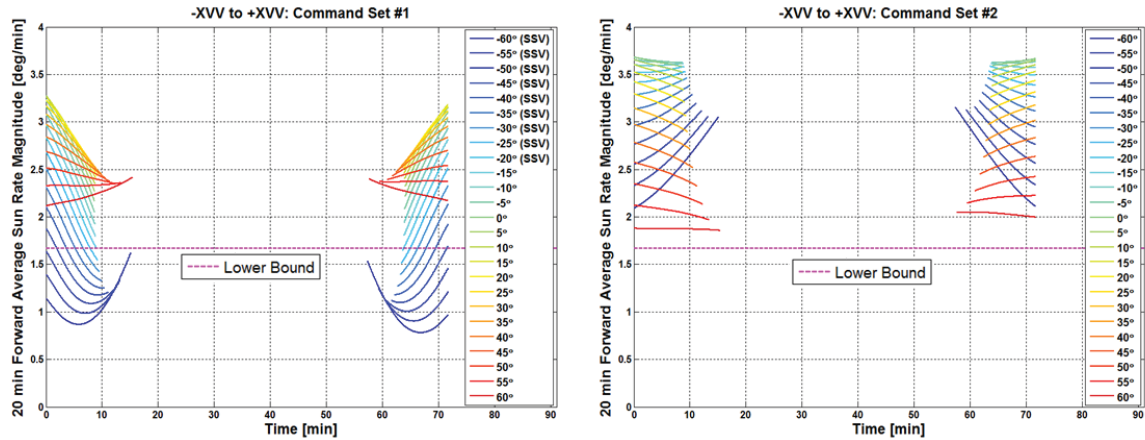

Figure 10. Comparison of average sun rate magnitude for various solar beta angles for –XVV to +XVV command set #1 (left) and the TCOPM command set #2 (right).

CONCLUSION

Part of the thermal analysis for ISS maneuvers is to check for a static sun vector, which may indicate excessive sun exposure to ISS hardware. If an SSV is observed, a more in-depth analysis must be performed to ensure the safety of the ISS structure. The initial OPMs had the potential for an SSV and thus had to be screened carefully each time they were used. The recently designed Thermally-Constrained OPMs presented in this paper do not encounter SSVs and have success-

fully been used on-orbit since 2013. These two TCOPMs are now used exclusively each time a transition between ±XVV attitudes is necessary, resulting in significant propellant savings without endangering the ISS.

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